What you'll Learn About

- How to find the derivative of:
- Functions with positive and negative integer powers
- Functions with products and quotients

A) Using a definition of the derivative find the derivative of $y = x^2$ at $x = a$

$$f'(a) = \lim_{x \to a} \frac{x^2 - a^2}{x - a} = \frac{(x+a)(x-a)}{x-a} = x + a = 2a$$

$f'(a) = 2a$

$f(x) = x^2 \quad f'(x) = 2x$

B) Using a definition of the derivative find the derivative of $y = x^3$ at $x = a$

$$f'(a) = \lim_{x \to a} \frac{x^3 - a^3}{x - a} = \frac{(x-a)(x^2 + ax + a^2)}{x-a} = x^2 + ax + a^2$$

$f'(a) = a^2 + a \cdot a + a^2$

$f'(a) = 3a^2$

$f(x) = x^3 \quad f'(x) = 3x^2$

C) Using a definition of the derivative find the derivative of $y = x^2 + 4$ at $x = a$

$$\lim_{x \to a} \frac{x^2 + 4 - (a^2 + 4)}{x - a} = \frac{x^2 - a^2}{x - a} = 2a$$

drop the power out in front and decrease the power by 1

Find the derivative using the power rule

$\frac{d}{dx}(\text{constant}) = 0$

↑ derivative with respect to x

D) $f(x) = 3 + x^2 - x^3 + x^5$

$f'(x) = 0 + 2x - 3x^2 + 5x^4$

E) $y = \frac{x^4}{5} + 3x^7$

$y = \frac{1}{5}x^4 + 3x^7$

$\frac{dy}{dx} = y' = \frac{4}{5}x^3 + 21x^6$

The derivative of y with respect to x

F) $y = x^{-3}$

$\frac{1}{x} = x^{-1}$

$y = x^{-3} = \frac{1}{x^3}$

$\frac{dy}{dx} = -3x^{-4} = \frac{-3}{x^4}$

G) $y = \frac{x^{-5}}{3} + \frac{x^{-3}}{4} - \frac{1}{x}$

$y = \frac{1}{3}x^{-5} + \frac{1}{4}x^{-3} - x^{-1}$

$\frac{dy}{dx} = \frac{-5}{3}x^{-6} - \frac{3}{4}x^{-4} + x^{-2}$

$\frac{dy}{dx} = \frac{-5}{3x^6} - \frac{3}{4x^4} + \frac{1}{x^2}$

$\sqrt{x} = x^{1/2}$

$\sqrt[3]{x} = x^{1/3}$

H) $f(x) = 4\sqrt{x} - \frac{1}{x} + \frac{2}{\sqrt{x}}$

$f(x) = 4x^{1/2} - \frac{1}{x} + \frac{2}{x^{1/2}}$

$f(x) = 4x^{1/2} - 1x^{-1} + 2x^{-1/2}$

$f'(x) = 2x^{-1/2} + x^{-2} - 1x^{-3/2} = \frac{2}{\sqrt{x}} + \frac{1}{x^2} - \frac{1}{\sqrt{x^3}}$

$f(x) = 1x^1$

$f'(x) = 1x^0 = 1$

Find the Horizontal Tangents of each curve

Horizontal Tangent

Max | Min

1) Find $\frac{dy}{dx}$

2) Set $\frac{dy}{dx} = 0$

I) $y = x^3 + 2x^2$

$\frac{dy}{dx} = 3x^2 + 4x$

$0 = 3x^2 + 4x$

$0 = x(3x + 4)$

$x = 0 \quad x = -\frac{4}{3}$

(0, 0)

$(-\frac{4}{3}, \quad)$

J) $y = \frac{2}{3}x^3 - \frac{5}{2}x^2 - 3x$

$\frac{dy}{dx} = 2x^2 - 5x - 3$

$0 = 2x^2 - 5x - 3$

$0 = (2x + 1)(x - 3)$

$x = -\frac{1}{2} \quad x = 3$